Mechanika III - 8.12.18

Metody perturbacyjne

$$x = x_0 + \varepsilon x_1 + \boxed{\varepsilon^2 x_2 + O(\varepsilon^3)}$$

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$$x = x_0 + \varepsilon x_1$$

$$\ddot{x} + \omega_0^2 x + \varepsilon x^3 = 0$$

$$\varepsilon^0: \ \ddot{x}_0 + \omega_0^2 x_0 = 0 \longrightarrow x_0 = a \cdot \sin \omega_0 t$$

$$\varepsilon^1: \ \ddot{x}_1 + \omega_0^2 x_1 + \varepsilon x_0^3 = 0$$

$$= -\varepsilon a^3 \sin^3 \omega_0 t =$$

$$\varepsilon^1: \ \ddot{x}_1 + \omega_0^2 x_1 = -\varepsilon a^3 \left(\frac{3}{4} \sin \omega_0 t - \frac{1}{4} \sin 3\omega_0 t\right)$$

Metoda Galerkina

$$L(x) = \ddot{x} + \omega_0^2 x + \varepsilon x^3$$

x - rozwiązanie dokładne

$\tilde{x}$ - rozwiązanie przybliżone

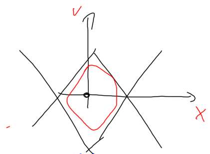
$$L(x) = 0 \quad \approx$$

$$L(\tilde{x}) = l(t)$$

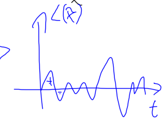
$$x = \sum_{i=1}^{+\infty} a_i \sin \omega_i t$$

$$x = \sum_{i=1}^{+\infty} a_i \sin\omega_i t$$

$$\tilde{x} = \sum_{i=1}^{N} \tilde{a}_i \sin\omega_i t$$



$$L(\tilde{x}) - L(x) \longrightarrow$$



$$\min \int_{(T)} \left(L(\tilde{x}) - L(x) \right)^2 dt$$

$$f_{śr} = \frac{\int_0^T f(t)\,dt}{T}$$

$$\mathcal{T} = \int_{(T)} L(\tilde{x})^2 dt$$

$$\min \mathcal{T}$$

dla $j=1,\dots,N$

$$0 = \frac{\partial \mathcal{T}}{\partial a_j} = \frac{\partial}{\partial a_j} \int_{(T)} \left[L\left(\sum_{i=1}^{N} a_i \sin\omega_i t \right) \right]^2 dt$$

$$\frac{\partial}{\partial a_j} \left(L\left(\sum_{i=1}^{N} a_i \sin\omega_i t \right)^2 \right) =$$

$$= 2 \cdot L\left(\sum_{i=1}^{N} a_i \sin\omega_i t \right) \cdot \frac{\partial}{\partial a_j} \left(\sum_{i=1}^{N} a_i \sin\omega_i t \right) =$$

$$= 2 \cdot L(\tilde{x}) \cdot \sin\omega_j t$$

dla $j=1,\dots,N$ $\int_{(T)} 2L(\tilde{x}) \cdot \sin\omega_j t \, dt = 0$

$$\int_{(T)} L(\tilde{x}) \cdot \sin\omega_j t \, dt = 0$$

Ostatecznie

dla $j=1,\dots,N$ $\int_{(T)} L\left(\sum_{i=1}^{N} a_i \sin\omega_i t \right) \cdot \sin\omega_j t \, dt = 0$

Równanie nieliniowego oscylatora

$$L(x) = \ddot{x} + \omega_0^2 x + \varepsilon x^3 = 0$$

$$\tilde{x} = a \sin \omega t = 0$$

$$\int_{(T)} L(a \sin \omega t) \cdot \sin \omega t \, dt = 0$$

$$\int_{(T)} \left(-a\omega^2 \sin \omega t + \omega_0^2 a \sin \omega t + \varepsilon a^3 (\sin \omega t)^3\right) dt =$$

$$= \int_{(T)} \left[\left(\omega_0^2 a - \omega^2 a + \varepsilon a^3 \cdot \frac{3}{4}\right) \cdot \sin \omega t - \frac{1}{4} \varepsilon a^3 \sin 3\omega t\right] \sin \omega t \, dt =$$

$$\sin^3 \omega t = \frac{3}{4} \sin \omega t - \frac{1}{4} \sin 3\omega t$$

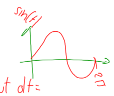
$$= \int_{(T)} \left(\omega_0^2 a - \omega^2 a + \varepsilon a^3 \cdot \frac{3}{4}\right) \sin^2 \omega t \, dt +$$

$$+ \int_{(T)} -\frac{1}{4} \varepsilon a^3 \cdot \sin 3\omega t \sin \omega t \, dt = 0$$

$$\oplus \begin{cases} \sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \sin\beta \cos\alpha \\ \sin(\alpha-\beta) = \sin\alpha \cos\beta - \sin\beta \cos\alpha \end{cases}$$

$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2 \sin(\alpha) \cos(\beta)$$

$$\sin(\alpha) \cdot \cos(\beta) = \frac{\sin(\alpha+\beta) + \sin(\alpha-\beta)}{2}$$



$$\int_{(T)} \left(\omega_0^2 a - \omega^2 a + \varepsilon \frac{3}{4} a^3\right) \sin^2 \omega t \, dt =$$

niech w oczy kole

$$= \left(\omega_0^2 a - \omega^2 a + \varepsilon \frac{3}{4} a^3\right) \frac{\pi}{\omega} = 0$$

$$\int_0^{\frac{2\pi}{\omega}} \sin^2 \omega t \, dt = \int_0^{\frac{2\pi}{\omega}} \left(\frac{1}{2} - \frac{1}{2} \cos 2\omega t\right) dt = \frac{1}{2} t \Big|_0^{\frac{2\pi}{\omega}} = \frac{\pi}{\omega}$$

$$\omega_0^2 a - \omega^2 a - \varepsilon \frac{3}{4} a^3 =$$

$$\omega^2 a = \omega_0^2 a - \varepsilon \frac{3}{4} a^3 \quad / a$$

$$\omega^2 = \omega_0^2 - \varepsilon \frac{3}{4} a^2$$

Drgania wymuszone układu nieliniowego

$$\ddot{x}+\omega_0^2x+\varepsilon x^3=f\sin\nu t$$

$$L(x)=\ddot{x}+\omega_0^2x+\varepsilon x^3-f\sin\nu t$$

$$x=a\sin\nu t$$

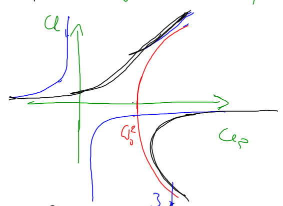
$$\int_{(T)}\left(-a\nu^2\sin\nu t+\omega_0^2a\sin\nu t+\varepsilon a^3(\sin\nu t)^3-f\sin\nu t\right)\sin\nu t\,dt=0$$

$$\int_{(T)}\left(\omega_0^2a-a\nu^2+\varepsilon a^3\cdot\frac{3}{4}-f\right)\sin^2\nu t\,dt+$$

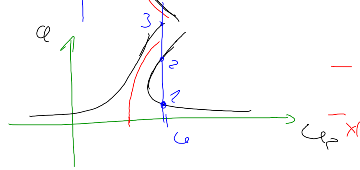
$$+\int_{(T)}(\ldots\ldots)\sin3\nu t\sin\nu t\,dt=0$$

$$\omega_0^2a-a\nu^2+\varepsilon a^3\cdot\frac{3}{4}-f=0\,/a$$

$$\omega_0^2-\nu^2+\varepsilon a^2\frac{3}{4}-\frac{f}{a}=0$$



$$\nu^2=\underbrace{\boxed{\omega_0^2+\varepsilon a^2\frac{3}{4}}}_{\omega_p}+\left(-\frac{f}{a}\right)$$



$-\omega_0^2(a)$

$-x_0\sin\omega_0t+$
$+\sin\boxed{\omega_0}t$

$-x_1(t)$ są ograniczone

- istnienie wielu rozwiązań