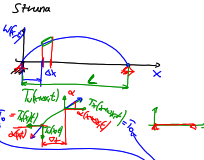


Drgania układów ciągłych



Struna

$$\sum \vec{F}_n: T_0 \sin \alpha - T_1 \cos \alpha = 0$$

$$\sum F_n: T_0(x+\Delta x, t) - T_1(x, t) = \Delta m \frac{\partial^2 u}{\partial t^2}$$

$$\Delta m = \rho \Delta x$$

$$\frac{\partial u}{\partial t} = \frac{\partial T_0}{\partial x} \Delta x$$

$$\frac{\partial^2 u}{\partial t^2} = T_0 \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{T_0}{\rho} \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial u}{\partial t} = a \frac{\partial u}{\partial x}$$

$$\frac{1}{2} \frac{m}{s} = \frac{m}{s} \frac{1}{s}$$

Wynagrodzić postać drgań

$w(x, t) = 0$
 $w(0, t) = 0$
 $w(l, t) = 0$
 $w(x, t) = X(x)T(t)$

$T''(t) + \omega^2 T(t) = 0$
 $\frac{1}{\omega^2} T'' = -\frac{X''}{X} = \text{const}$

$X'' + k^2 X = 0$

$X(x) = A \sin kx + B \cos kx$

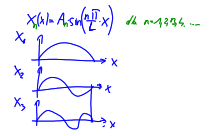
$0 = w(0, t) = X(0)T(t) = X(0) = 0$

$0 = w(l, t) = X(l)T(t) = X(l) = 0$

$\delta = X(0) = A \cdot 0 + B \cdot 1 = B = 0$

$0 = X(l) = A \sin kl$

$k_n l = n\pi \quad k_n = \frac{n\pi}{l} \quad n = 1, 2, 3, \dots$



Częstotliwości własne

$\frac{1}{\omega^2} \frac{T''}{T} = -\frac{X''}{X} = -\left(\frac{n\pi}{l}\right)^2$

$\frac{T''}{T} = -\omega^2 \left(\frac{n\pi}{l}\right)^2$

$T'' + \left(\frac{n\pi}{l}\right)^2 T = 0 \quad \text{dla } n=1, 2, 3, \dots$

$\ddot{z} + \omega_n^2 z = 0 \quad \omega_{n, \text{str}} = n \frac{\pi}{l} a$

