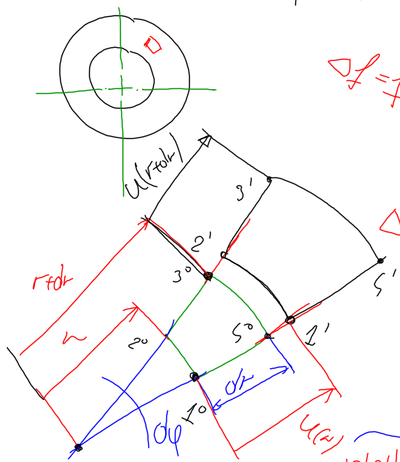
Wykład VI – 15.12.18

TEORIA SPRĘŻYSTOŚCI

Płaski osiowosymetryczny stan naprężenia



$\Delta f = f(x+\Delta x) - f(x) =$, $\Delta x \approx 0$

$\Delta f = f' \cdot \Delta x$

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{l' - l_0}{l_0}$$

l': $|2'3'|$, $|2°3°|$, l_0

$$\varepsilon_r = \frac{u(r+dr) + r + dr - (r + u(r)) - dr}{dr} = \frac{u(r+dr) - u(r)}{dr} = \frac{\frac{du(r)}{dr} \cdot dr}{dr} = \frac{du}{dr}$$

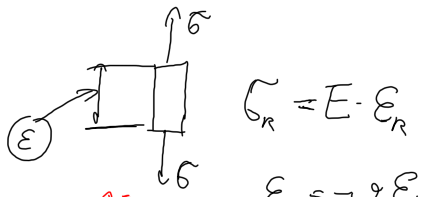
$|1'2'| = l'$, $|1°2°| = l°$

$$\varepsilon_\varphi = \frac{(u(r)+r)\,d\varphi - r\,d\varphi}{r\,d\varphi} = \frac{u(r)\,d\varphi}{r\,d\varphi} = \frac{u(r)}{r}$$

$$\varepsilon_r = \frac{du(r)}{dr}$$

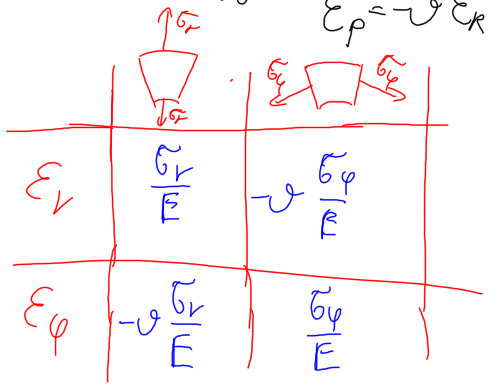
$$\varepsilon_\varphi = \frac{u(r)}{r}$$

Prawo Hooke'a



$\sigma_R = E \cdot \varepsilon_R$

$\varepsilon_\rho = -\nu \varepsilon_R$



$$\begin{cases} \varepsilon_r = \frac{1}{E}\left(\sigma_r - \nu \sigma_\varphi\right) \\ \varepsilon_\varphi = \frac{1}{E}\left(-\nu \sigma_r + \sigma_\varphi\right) \end{cases}$$

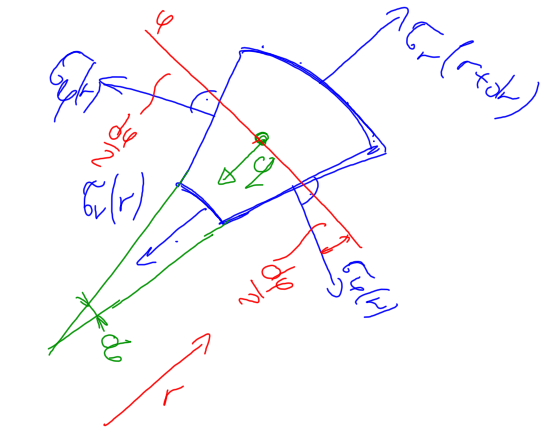
$$\begin{cases} \sigma_r = \frac{E}{1+\nu^2}\left(\nu \varepsilon_\varphi + \varepsilon_r\right) \\ \sigma_\varphi = \frac{E}{1+\nu^2}\left(\varepsilon_\varphi + \nu \varepsilon_r\right) \end{cases}$$

$$A = \begin{bmatrix} 1 & -\nu \\ -\nu & 1 \end{bmatrix}$$

$$A_r = \begin{bmatrix} 1 & -\nu \\ 1 & 1 \end{bmatrix}$$

$$A_\varphi = \begin{bmatrix} -\nu & 1 \\ 1 & 1 \end{bmatrix}$$

Statyka



$$\sigma_r(r+dr)(r+dr)d\varphi - \sigma_r(r) r d\varphi - q_r r d\varphi dr - 2\sigma_\varphi dr \sin\frac{d\varphi}{2} = 0$$

$$\sin\frac{d\varphi}{2} \cong \frac{d\varphi}{2}$$

$$\left(\sigma_r(r+dr)(r+dr) - \sigma_r r\right) d\varphi - q_r r d\varphi dr - \sigma_\varphi d\varphi dr = 0$$

$$\frac{d}{dr}\left(\sigma_r \cdot r\right) \cdot dr d\varphi - \sigma_\varphi d\varphi dr - q_r r d\varphi dr = 0$$

$$\frac{d}{dr}\left(\sigma_r \cdot r\right) - \sigma_\varphi = q_r \cdot r$$

$$\frac{d}{dr}(\sigma_r \cdot r) - \sigma_\varphi = q \cdot r$$

$$\sigma_r = \frac{E}{1+\vartheta^2}\left(\varepsilon_r'' + \vartheta\,\varepsilon_\varphi''\right), \qquad \varepsilon_r'' = \frac{du(r)}{dr}, \quad \varepsilon_\varphi'' = \frac{u(r)}{r}$$

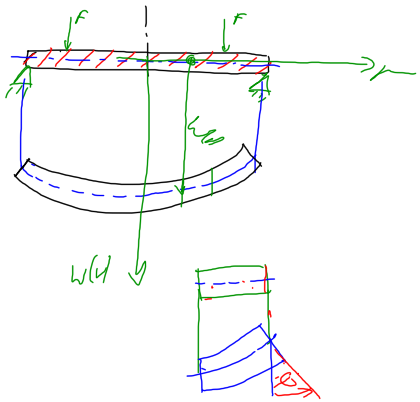
$$\sigma_\varphi = \frac{E}{1+\vartheta^2}\left(\vartheta \cdot \frac{du(r)}{dr} + \frac{u(r)}{r}\right)$$

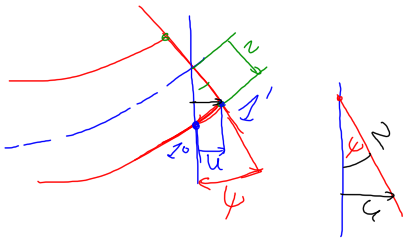
$$\frac{d}{dr}\left(\frac{E}{1+\vartheta^2}\left(r\cdot\frac{du}{dr} + \vartheta u\right)\right) - \frac{E}{1+\vartheta^2}\left(\vartheta\frac{du}{dr} + \frac{u}{r}\right) = q \cdot r$$

$$\frac{E}{1+\vartheta^2}\left[\left(\frac{du}{dr} + r\frac{d^2u}{dr^2} + \vartheta\frac{du}{dr}\right) - \left(\vartheta\frac{du}{dr} + \frac{u}{r}\right)\right] = q \cdot r$$

$$r\frac{d^2u}{dr^2} + \frac{du}{dr} - \frac{u}{r} = \frac{q\cdot r}{D_R} \qquad D_R = \frac{E}{1+\vartheta^2}$$

ZGINANIE PŁYT KOŁOWO SYMETRYCZNYCH





$$\frac{u}{z} = \sin\psi \cong \psi$$

$$u = z\psi$$

$$\varepsilon_r = z \cdot \frac{d\psi(r)}{dr}$$

$$\varepsilon_\varphi = \frac{z\psi(r)}{r}$$